arXiv:hep-ph/0210110 v1 7 Oct 2002

The invisible axion in a Randall-Sundrum universe

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(Dated: January 22, 2003)

We study the problem of integrating an invisible axion into the Randall-Sundrum scenario as an example of the difficulty of generating energy scales between the extremes of the Planck mass and the electroweak scale without unnatural fine tunings. In this scenario, the axion corresponds to the phase of complex bulk scalar field. We find that for simple bulk mechanisms to produce an energy scale f associated with $U(1)_{\rm PQ}$ breaking requires a fine tuning of the order of f/M_4 or TeV$/f$, where M_4 is the 4d Planck mass. The AdS/CFT correspondence suggests that such fine tunings should occur generically when we attempt to introduce intermediate energy scales into the Randall-Sundrum picture.

PACS numbers: 11.10.Kk, 14.80.Mz, 04.50.+h, 11.30.Er

I. INTRODUCTION

One of the features which the Randall-Sundrum scenario [1] shares with other solutions to the hierarchy problem is that it assumes a desert between the scale of electroweak physics and the scale of gravity. Although new phenomena—strong gravity and bulk Kaluza Klein modes—appear above the electroweak scale, all the physics in this scenario can be expressed in terms of these two scales. Such a picture is usually adequate since we have no direct evidence of phenomena between these energies. Yet in some cases we may wish to introduce some new physics whose dynamics occur at an intermediate scale. The difficulty in the Randall-Sundrum brane world is to understand how a scale can naturally arise, surviving in the low energy theory, that is not either of these two natural extremes.

A specific instance of where such an intermediate scale is needed occurs in the invisible axion solution to the strong CP problem [2]. The vacuum structure of QCD combined with the CP violation in the weak interactions permits an interaction of the form

$$\bar\theta \frac{\alpha_s}{8\pi} {\rm Tr}[\epsilon_{\mu\nu\lambda\rho}F^{\mu\nu}F^{\lambda\rho}] \qquad (1.1)$$

where $F^{\mu\nu}$ is the QCD field strength. This interaction violates P and CP and is highly constrained by measurements of the neutron dipole moment which require $\bar\theta \le 10^{-9}$. As a free parameter, $\bar\theta$ must thus be finely tuned for an acceptable theory. Peccei and Quinn [3] showed that if $\bar\theta$ is promoted to a dynamical field $a(x^\mu)$ which is the Goldstone boson associated with a spontaneously broken global $U(1)_{\rm PQ}$ symmetry, then $\bar\theta$ is dynamically driven to zero. Although this field, the axion, is a Goldstone mode, it does acquire a mass of the order $\Lambda^2_{\rm QCD}/f$ where f is scale at which the $U(1)_{\rm PQ}$ breaking occurs. An f of the order of the electroweak scale produces too massive an axion for experimental constraints.

An acceptable axion mass does occur when the $U(1)_{\rm PQ}$ breaks at some high scale $f \gg$ TeV. From astrophysical observations, this scale should lie within the interval [4, 5]

$$10^{10}\,{\rm GeV} \lesssim f \lesssim 10^{13}\,{\rm GeV}. \qquad (1.2)$$

Since the invisible axion models do not address the hierarchy problem, they do not attempt to explain whether such a scale can arise naturally.

In the Randall-Sundrum scenario, the only natural scales are the bulk Planck mass, M_5, and the AdS curvature, k. Other, exponentially smaller scales do arise when the physics responsible for them is confined to a region at some distance from the UV brane. Although the mass scales for the fields confined to the IR brane are also of the order M_5, when the fields there are rescaled to remove red-shift factors introduced by the induced metric on the brane, the apparent mass scales on the IR brane can be naturally of the order of the electroweak scale. Goldberger and Wise [6] showed that the position of the IR brane relative to the UV brane can be stabilized—and thus the electroweak-gravity hierarchy—without finely turning the parameters of the stabilization mechanism. The observed Planck mass in low energy, four dimensional effective theory, determined by $M_4^2 \approx M_5^3/k$, remains large.

In this article we shall use the invisible axion as a case study of the difficulty in introducing new scales into the Randall-Sundrum scenario. For this purpose it provides an ideal subject—the scale f is experimentally constrained to not be that associated with either of the branes. These constraints arise, moreover, from low energy physics with respect to the electroweak scale so that bulk effects do not allow us to modify these bounds as in scenarios with large extra dimensions [7]. Here we shall see that straightforward attempts to produce an intermediate scale TeV $\ll f \ll M_4$ all require some fine tuning which is not significantly better than simply assuming $\bar\theta < 10^{-9}$.

The next section discusses the origin of the scale f associated with the breaking of $U(1)_{\rm PQ}$ when the axion is the phase of a complex bulk scalar field. In Section III, we show that for natural bulk potentials, such as a

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mass term or a potential well, some fine tuning is needed to obtain an acceptable f. Introducing a third brane in the bulk, as in section IV would also require some fine tuning in its stabilization. The necessity of some fine tuning has a natural explanation in terms of the AdS/CFT correspondence, presented in Section V. Section VI concludes.

II. THE INVISIBLE AXION AS A BULK FIELD

The action for the original Randall-Sundrum model contains an Einstein-Hilbert term and cosmological constant for the bulk as well as tension terms for the branes

$$\begin{aligned} S_{\rm RS} &= M_5^3 \int d^4x dy \sqrt{-g}\, [-2\Lambda + R] \\ &+ M_5^3 \int_{\rm UV} d^4x\, \sqrt{-h_0}\, [-2\sigma_0 + 4K_0] \\ &+ M_5^3 \int_{\rm IR} d^4x\, \sqrt{-h_1}\, \left[-2\sigma_1 + 4K_1 + M_5^{-3}\mathcal{L}_{\rm sm}\right]. \end{aligned} \tag{2.1}$$

Here $h_{0,1}$ and $K_{0,1}$ are the determinant of the induced metric and the trace of the extrinsic curvature on the UV and IR branes. In terms of the AdS curvature k, the cosmological constant is $\Lambda = -6k^2$ and the brane tensions should be $\sigma_0 = -\sigma_1 = 6k$. $\mathcal{L}_{\rm sm}$ represents the standard model Lagrangian. The UV and IR branes are located at $y = 0$ and $y = \Delta y$ respectively.

To solve the strong CP problem in the low energy theory, we introduce a global $U(1)_{\rm PQ}$ symmetry under which the brane quark and Higgs fields transform non-trivially [2]. Since the scale of Peccei-Quinn symmetry breaking does not lie near the scales associated with either brane, it is natural to attempt to break this symmetry through bulk dynamics. Thus, the axion will correspond to the phase of a bulk complex scalar field,

$$\sigma = \frac{\rho}{\sqrt{2}} e^{ia}. \tag{2.2}$$

The dynamics of this field will be determined by a $U(1)_{\rm PQ}$-symmetric potential,

$$\begin{aligned} S_\sigma &= \int d^4x dy \sqrt{-g}\, \left[-\nabla_a\sigma^\dagger\nabla^a\sigma - V(\sigma^\dagger\sigma)\right] \\ &+ \int_{\rm UV} d^4x\, \sqrt{-h_0}\, \mathcal{V}_0(\sigma^\dagger\sigma) \\ &+ \int_{\rm IR} d^4x\, \sqrt{-h_1}\, \mathcal{V}_1(\sigma^\dagger\sigma), \end{aligned} \tag{2.3}$$

Here, as in Goldberger and Wise [6], the potentials on the branes will be used to fix the value of the field ρ on the UV and IR branes to be respectively $\rho_0 M_5^{3/2}$ and $\rho_1 M_5^{3/2}$.

The form of the $U(1)_{\rm PQ}$ symmetry breaking due to the bulk potential $V(\rho)$ is quite different from the invisible axion solution in $3+1$ dimensions. A vacuum solution in which $\rho = f$ for the bulk theory would necessarily require some fine tuning of the bulk potential to obtain a realistic f. Instead, ρ can have some non-trivial dependence on the extra dimension which also breaks the $U(1)_{\rm PQ}$. In going to the low energy effective theory, integrating out the bulk field ρ will induce a scale f for the axion which plays the same role as the symmetry breaking scale in the 4d invisible axion models.

The field equations for the components of σ are

$$\begin{aligned} \nabla^2 a + \frac{2}{\rho}\nabla_a\rho\nabla^a a &= 0 \\ \nabla^2\rho - \frac{\delta V}{\delta\rho} - \rho\nabla_a a\nabla^a a &= 0. \end{aligned} \tag{2.4}$$

The important dynamics of the axion occurs at energies well-below the TeV scale beyond which bulk effects become important. In this low energy regime, we shall neglect the higher-order Kaluza-Klein modes of the axion which, since it is a Goldstone mode, will have a massless mode which remains in the effective theory. Thus we shall consider only the lowest mode in the Kaluza-Klein tower, $a \to a(x^\mu)$. This situation differs greatly from a bulk axion in models with large, flat extra dimensions where the Kaluza-Klein modes of the axion are of the order of the inverse compactification radius and are important in the low energy ($\ll$ TeV) theory [7]. The field ρ is not protected by any symmetry and its vacuum state is determined by the bulk potential $V(\rho)$ so we shall neglect any x^μ-dependent fluctuations about the vacuum configuration, $\rho = \rho(y)$, as small in the effective theory,

$$\rho'' - 4k\rho' \approx \frac{\delta V}{\delta\rho} \qquad\qquad \partial_\mu\partial^\mu a \approx 0. \tag{2.5}$$

The axion from this perspective becomes a massless field while the scalar field ρ has its dynamics set by the scale of the bulk physics. At energies below a TeV, we can integrate out $\rho(y)$ to obtain an effective description of the axion dynamics,

$$\begin{aligned} S_\sigma &= \int d^4x \left[-\frac{1}{2}\left(\int_0^{\Delta y} dy\, 2e^{-2ky}\rho^2(y)\right)\partial_\mu a\partial^\mu a\right] \\ &= \int d^4x\, \left[-\tfrac{1}{2}f^2\partial_\mu a\partial^\mu a + \cdots\right], \end{aligned} \tag{2.6}$$

where we have defined

$$f^2 \equiv \int_0^{\Delta y} dy\, 2e^{-2ky}\rho^2(y) \tag{2.7}$$

which sets the scale associated with the axion by rescaling

$$a(x^\mu) \to \frac{a(x^\mu)}{f}. \tag{2.8}$$

After this rescaling, the axion has the proper dimensions for a scalar field in the 4d effective theory.

The remaining components needed to implement a solution to the the strong CP problem closely resemble standard invisible axion models. Typically such models introduce heavy quarks which carry $U(1)_{\rm PQ}$ charge and couple to σ—KSVZ axions [8]—or an extra Higgs doublet is added which couples to σ—DFSZ axions [9]. To obtain the the latter model within a Randall-Sundrum scenario, we add an interaction between a pair of brane Higgs doublets, Φ_1 and Φ_2, and the bulk complex field σ,

$$S_{\rm int} = \int_{\rm IR} d^4x\, \sqrt{-h_1}\, \left[\kappa M_5^{-1}\epsilon_{ij}\Phi_1^i\Phi_2^j(\sigma^\dagger(\Delta y))^2 + {\rm h.c.}\right]; \tag{2.9}$$

here we have extracted a factor of the Planck mass so that κ is a dimensionless coupling. Using that on the IR brane, $\sigma(\Delta y) = \frac{1}{\sqrt{2}}\rho_1 M_5^{3/2} e^{ia}$, and rescaling the axion using Eq. (2.8) and the Higgs fields by $\Phi_{1,2} \to e^{k\Delta y}\Phi_{1,2}$ so that they have canonically normalized kinetic terms, the leading behavior from Eq. (2.9) in the low energy limit is

$$S_{\rm int} = \int d^4x\, \left[\kappa_{\rm eff}\epsilon_{ij}\Phi_1^i\Phi_2^j e^{-2ia/f} + {\rm h.c.}\right], \tag{2.10}$$

where

$$\kappa_{\rm eff} \equiv \tfrac{1}{2}\kappa\rho_1^2(e^{-k\Delta y}M_5)^2 \sim \mathcal{O}({\rm TeV}^2). \tag{2.11}$$

The standard model fields confined to the IR brane also have $U(1)_{\rm PQ}$ charges which we shall choose to be $+\frac{1}{2}$ for the right-handed fermions and $-\frac{1}{2}$ for the left-handed $SU(2)$-doublet fermions. With these assignments, the Higgs fields have $U(1)_{\rm PQ}$ charges $+1$ so that Eq. (2.10) is an invariant interaction. The fact that the Higgs fields transform non-trivially under $U(1)_{\rm PQ}$ allow some of their degrees of freedom to mix with massless mode in the effective theory that arises when we integrate out the extra dimension (2.6). At this point the theory is essentially indistinguishable from 4d invisible axion model.

III. BULK POTENTIALS

The symmetry breaking scale in Eq. (2.7) will be large unless the scalar field can be excluded from the region near the Planck brane. If $\rho \ll 1$ until some intermediate position y_a, then the warping factor from the bulk metric will yield an f of the order $e^{-ky_a}M_4$. We can obtain some intuition as to the necessary form for the bulk potential by noting that the field equation for ρ,

$$\rho'' - 4k\rho' = \frac{\delta V}{\delta\rho}, \tag{3.1}$$

is that of a particle rolling in the inverted potential, $-V(\rho)$, under the influence of a *negative* friction term. If the particle starts at $\rho = 0$ with a small initial velocity, it tends to accelerate. Thus, in regions where the potential is approximately constant, its value will be exponentially larger after a finite interval or, conversely, throughout most of the interval $\rho(y)$ will be exponentially small. This evolution should not occur throughout the entire bulk since then the integral (2.7) would then only produce an $f \sim e^{-k\Delta y}M_4 \sim$ TeV. If $V(\rho)$ decreases substantially after ρ has become sufficiently large, this change will act to dissipate the 'kinetic energy' produced by the friction term and ρ will grow more slowly so that ρ is not exponentially weighted toward that latter end of this stage of its evolution in the bulk. After this dissipative stage, we could follow it with another region in which $V(\rho)$ is approximately flat—as long as ρ does not grow exponentially larger than its values during the prior stage before it reaches the IR brane, the exponential factor in Eq. (2.7) insures that the integral will be dominated by intermediate values of y.

A. A free massive bulk field

The simplest potential is a mass term for the bulk scalar. From the preceding arguments, a positive mass squared term will have the effect of accelerating the growth of the field already produced by the friction term as we move from the UV to the IR brane. Although it might seem that a negative mass term could slow the effects of the friction term, we shall see that this case also does not produce an acceptable value for f without some fine tuning. Since the field equations for this potential can be solved exactly, we present both cases.

Consider a generalized mass term, to allow for either a stable or unstable extremum at $\rho = 0$,

$$V(\rho) = \tfrac{1}{2}(\mu - 4)k^2\rho^2, \tag{3.2}$$

where μ is a dimensionless parameter. When $\mu > 0$, the solution is

$$\rho(y) = \rho_1 M_5^{3/2}\frac{e^{2ky}}{e^{2k\Delta y}}\frac{\sinh(\sqrt{\mu}ky)}{\sinh(\sqrt{\mu}k\Delta y)} \tag{3.3}$$

while for $\mu < 0$,

$$\rho(y) = \rho_1 M_5^{3/2}\frac{e^{2ky}}{e^{2k\Delta y}}\frac{\sin(\sqrt{-\mu}ky)}{\sin(\sqrt{-\mu}k\Delta y)}. \tag{3.4}$$

Here we have imposed the boundary conditions $\rho(0) = 0$ and $\rho(\Delta y) = \rho_1$. For $\mu > 0$, the scale f is always of the order

$$f/M_4 \sim \rho_1 e^{-k\Delta y} \tag{3.5}$$

which is too small for ρ_1 of a natural size, $\mathcal{O}(1)$. In fact ρ_1 should be somewhat small if the presence of the scalar field is not to distort the background AdS$_5$ geometry. For $\mu < 0$ the zeros of the denominator can generate much larger scales than (3.5); when $\sqrt{-\mu}k\Delta y = n\pi - \delta$ $(n = 1, 2, 3, \ldots)$,

$$f/M_4 \sim \rho_1\frac{\sqrt{-\mu}}{\sqrt{1-\mu}}\frac{e^{-k\Delta y}}{\delta}, \tag{3.6}$$

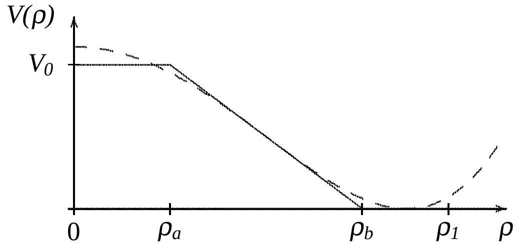

FIG. 1: We shall study the profile of a bulk field ρ whose potential is given by the toy model shown by the solid line. It can be regarded as a crude model for $0 \leq \rho \leq \rho_1$ of a quartic, double-well potential shown by the dashed line.

but only if we finely tune $\delta \lesssim 10^{-6}$.

Note that in either case, if we relax the requirement $\rho(0) = \rho_0 = 0$ then ρ_0 must itself be tuned to be of the order $\leq 10^{-6}$. In this case μ should also satisfy $\mu > 1$ since otherwise the large negative mass squared favors an exponentially large value of $\rho(y)$ within the bulk so that we would find $f \gg M_4$.

B. A potential well

The reason that a mass term alone does not succeed is that the potential contains no feature which might allow a brief growth of $\rho(y)$ which appears in the vicinity of $0 < y_a < \Delta y$ but which is damped soon after so that the field assumes an $\mathcal{O}(1)$ value at the IR brane.

To model this behavior with a potential which we can solve exactly, we shall study the following toy potential,

$$V(\rho(y)) = \begin{cases} V_0 & \text{for } \rho \leq \rho_a \\ V_0\left[1 - \frac{\rho(y)-\rho(y_a)}{\Delta\rho}\right] & \text{for } \rho_a \leq \rho \leq \rho_b \\ 0 & \text{for } \rho \geq \rho_b \end{cases} \tag{3.7}$$

where

$$\Delta\rho = \rho_b - \rho_a. \tag{3.8}$$

The parameters specifying this potential are ρ_a, ρ_b and V_0. We next shall estimate how carefully the form of the potential must be tuned to achieve an acceptable value for f.

Naïvely, (3.7) resembles a portion of a double well potential seen in the vicinity of the origin, as shown in Fig. 1, and we assume that $V(-\rho) = V(\rho)$. Note that $V(\rho)$ could grow again for larger values of ρ, but as long as this growth occurs for values $\rho(y) > \rho_1$, it will not affect our derivation. Note also that we are implicitly assuming that $\rho(y)$ is a monotonically increasing function of y, which occurs provided the well is not so deep that all the 'kinetic energy' is dissipated and the particle rolls back toward $\rho = 0$. It is also important that $\rho(0) = 0$ at the UV brane since it is difficult for any natural potential to suppress it quickly enough to prevent the small y region from dominating Eq. (2.7). Fortunately, since we have assumed that the brane potentials in Eq. (2.3) are $U(1)_{\rm PQ}$ symmetric, as long as they are analytic functions of ρ, $\rho = 0$ will be an extremum on the branes so we do not need to fine tune $\rho(0) = 0$ to be a minimum. We shall assume hereafter that $\rho_0 = 0$ and $\rho_1 \approx \mathcal{O}(1)$.

Let us define positions $y_a < y_b$ such that $\rho(y_a) = \rho_a$ and $\rho(y_b) = \rho_b$. The solution to Eq. (3.1) for this toy potential with the boundary conditions $\rho(0) = 0$ and $\rho(\Delta y) = \rho_1$ is then

$$\rho(y) = c\left(e^{4ky} - 1\right) \tag{3.9}$$

for $0 < y < y_a$,

$$\begin{aligned}\rho(y) &= c\left(e^{4ky} - 1\right) - \frac{1}{16k^2}\frac{V_0}{\Delta\rho}\left(e^{4k(y-y_a)} - 1\right) \\ &\quad + \frac{1}{4k}\frac{V_0}{\Delta\rho}(y - y_a)\end{aligned} \tag{3.10}$$

for $y_a < y < y_b$ and

$$\begin{aligned}\rho(y) &= c\left(e^{4ky} - 1\right) - \frac{1}{16k^2}\frac{V_0}{\Delta\rho}\left(e^{-4ky_a} - e^{-4ky_b}\right)e^{4ky} \\ &\quad + \frac{1}{4k}\frac{V_0}{\Delta\rho}(y_b - y_a)\end{aligned} \tag{3.11}$$

for $y_b < y$. For convenience we have defined the constant c to be

$$\begin{aligned}c &\equiv \frac{\rho_1 M_5^{3/2}}{e^{4k\Delta y} - 1} + \frac{1}{16k^2}\frac{V_0}{\Delta\rho}\left(e^{-4ky_a} - e^{-4ky_b}\right)\frac{e^{4k\Delta y}}{e^{4k\Delta y} - 1} \\ &\quad - \frac{1}{4k}\frac{V_0}{\Delta\rho}\frac{y_b - y_a}{e^{4k\Delta y} - 1}.\end{aligned} \tag{3.12}$$

To learn whether the potential requires any fine tunings, we can reparameterize the slope of the potential in terms of the natural scales available, k, M_5,

$$\frac{V_0}{\Delta\rho} \equiv 16k^2 M_5^{3/2}\alpha \tag{3.13}$$

where α should be some constant of order one. To leading order in powers of the exponential factors, the integral (2.7) is then

$$\begin{aligned}f^2/M_4^2 &= \tfrac{40}{3}\alpha^2 e^{-2ky_a} - \tfrac{32}{3}\alpha^2\left(1 + 3k(y_b - y_a)\right)e^{-2ky_b} - \tfrac{8}{3}\alpha^2 e^{-2k(y_b-y_a)}e^{-2ky_b} \\ &\quad + \tfrac{1}{3}\left(\rho_1^2 + 16\rho_1 k(y_b - y_a)\alpha - 128k^2(y_b - y_a)^2\alpha^2\right)e^{-2k\Delta y}\end{aligned}$$

$$-\tfrac{8}{3}\alpha\left(\rho_1 - 4k(y_b - y_a)\alpha\right)\left(1 - e^{-2k(y_b-y_a)}\right)e^{-2k(\Delta y - y_b)}e^{-2k\Delta y} + \cdots \qquad (3.14)$$

For $e^{-ky_a} \gg e^{-ky_b} \gg e^{-k\Delta y}$ we have

$$\frac{f}{M_4} \approx 2\sqrt{\tfrac{10}{3}}\alpha e^{-ky_a}. \qquad (3.15)$$

The chief contribution to (2.7) comes from the region $y \sim y_a$.

We can now show that to achieve a realistic value for y_a requires finely tuning the potential. In terms of the parameters y_a, y_b of the solution we have

$$\begin{aligned} \rho_a M_5^{-3/2} &= \alpha\left[1 - e^{-4k(y_b-y_a)}\right] + \cdots \qquad (3.16)\\ \rho_b M_5^{-3/2} &= 4k\alpha(y_b - y_a) \\ &\quad + e^{-4k(\Delta y - y_b)}\left[\rho_1 - 4k\alpha(y_b - y_a)\right] + \cdots. \end{aligned}$$

Note that ρ_a, ρ_b and α are the parameters specifying the shape of the potential. The first line in Eq. (3.16) indicates that we must tune $\alpha^{-1}\rho_a M_5^{-3/2} \approx 1$ to within a fractional correction of the order $e^{-4k(y_b-y_a)}$. We can evade this fine tuning if $y_b \sim y_a$; however, then we must finely tune the value of ρ_b to within an order $e^{-4k(\Delta y - y_b)}$ correction. From Eq. (3.15) and Eq. (1.2), $16 \lesssim ky_a \lesssim 23$ and an electroweak-Planck hierarchy of 10^{-16} requires $k\Delta y \sim 37$. Assuming $y_b \sim y_a$ yields then an exponentially small correction.

IV. AN INTERMEDIATE BRANE

The energy scale associated with the Standard Model fields remains naturally light since they are confined to the IR brane at which the redshift suppresses the strength gravity by an exponential factor. Similarly, the introduction of another brane, at some intermediate distance in the bulk, $0 < y_a < \Delta y$, will produce a new energy scale $e^{-ky_a}M_4$. Just as for the tensions of the UV and IR branes, the tension of this intermediate brane must be finely tuned in terms of the bulk cosmological constant to satisfy the Israel conditions at the brane. The model requires one fine tuning for each brane. Here we shall show that even when the positions of the branes are stabilized, one fine tuning beyond the Randall-Sundrum model remains.

Consider a bulk space-time in which the UV and IR branes reside as usual at the fixed points of the orbifold, $y = 0$ and $y = \Delta y$ respectively, while the intermediate brane partitions the bulk into two regions with cosmological constants $\Lambda_0 = -6k_0^2$ $(0 \le y \le y_a)$ and $\Lambda_0 = -6k_1^2$ $(y_a \le y \le \Delta y)$. Matching the induced metric on both sides of the axion brane, the bulk metric can be written in the form

$$ds^2 = e^{-2k_0 y}\,\eta_{\mu\nu}\,dx^\mu dx^\nu + dy^2 \qquad (4.1)$$

for $0 \le y \le y_a$ and

$$ds^2 = e^{-2k_1 y}e^{-2(k_0-k_1)y_a}\,\eta_{\mu\nu}\,dx^\mu dx^\nu + dy^2 \qquad (4.2)$$

for $y_a \le y \le \Delta y$. The Israel jump conditions across the branes require the UV, IR and axion branes to have tensions respectively of

$$\sigma_0 = 6k_0 \qquad \sigma_1 = -6k_1 \qquad \sigma_a = 3(k_0 - k_1). \qquad (4.3)$$

Note that when the cosmological constants are equal, $k_0 = k_1 = k$, the axion brane becomes a tensionless 'probe' brane.

Eq. (4.3) summarizes the three fine tunings necessary for this model. One of these fine tunings is equivalent to the vanishing of the cosmological constant in the low energy effective theory. As in the Randall-Sundrum scenario, we shall not attempt to resolve this fine tuning. In a scenario with a further extra dimension, this vanishing can be reduced to the tuning of the initial conditions rather than a tuning of the parameters in the gravitational action [10].

If the space-time ended at the axion brane, it would be possible to stabilize the radion mode corresponding to its distance from UV brane which would remove one of the fine tuning. However, since whatever stabilization mechanism is used must extend further, to the IR brane, we need to make a slightly less severe fine tuning, of the order $e^{-k(\Delta y - y_a)}$ to prevent it from significantly distorting the background geometry and overwhelming the stabilization of the IR brane. This fine tuning appears straightforwardly in a simple extension of the Goldberger-Wise mechanism [6].

For simplicity, we shall set the cosmological constants to be equal upon either side of the axion brane, $k_0 = k_1 = k$. To stabilize the axion brane, we add a real bulk scalar field to the scenario,

$$S_\phi = M_5^3 \int d^5x\,\sqrt{-g}\,\left[-\tfrac{1}{2}\nabla_a\phi\nabla^a\phi - \tfrac{1}{2}m^2\phi^2\right] \qquad (4.4)$$

with a mass $m = k\sqrt{\nu^2 - 4}$ (with $\nu = 2 + \epsilon$) and which assumes values v_0, v_a and v_1 on the UV, axion and IR branes respectively. We have extracted a factor of M_5^3 so that $\phi(y)$ is dimensionless. Although we wish only to stabilize the intermediate brane, the IR brane cannot in general be transparent with respect to this bulk field. Unless the slope of the field vanishes at $y = \Delta y$, it will develop a kink at this orbifold point so we must assume some dynamics on the IR brane to produce this kink. The solution in each of the two bulk regions is

$$\phi(y) = \begin{cases} \phi_-(y) & \text{for } 0 \le y \le y_a \\ \phi_+(y) & \text{for } y_a \le y \le \Delta y \end{cases} \qquad (4.5)$$

where

$$\begin{aligned}
\phi_-(y) &= -\frac{v_0 e^{(2-\nu)ky_a} - v_a}{1-e^{-2\nu k y_a}} e^{(2+\nu)k(y-y_a)} + \frac{v_0 - v_a e^{-(2+\nu)ky_a}}{1-e^{-2\nu k y_a}} e^{(2-\nu)ky} \\
\phi_+(y) &= -\frac{v_a e^{(2-\nu)k(\Delta y - y_a)} - v_1}{1-e^{-2\nu k(\Delta y - y_a)}} e^{(2+\nu)k(y-\Delta y)} + \frac{v_a - v_1 e^{-(2+\nu)k(\Delta y - y_a)}}{1-e^{-2\nu k(\Delta y - y_a)}} e^{(2-\nu)k(y-y_a)}. \qquad (4.6)
\end{aligned}$$

Integrating the scalar field action over the extra dimension produces an effective potential,

$$\begin{aligned}
\frac{V_{\rm eff}(y_a,\Delta y)}{2kM_5^3} &= \epsilon v_0^2 + 2(2+\epsilon)\left(v_0 e^{-\epsilon k y_a} - v_a\right)^2 e^{-4ky_a} + 2(2+\epsilon)\left(v_0 e^{-\epsilon k y_a} - v_a\right)^2 e^{-2\epsilon k y_a} e^{-8ky_a} \\
&\quad + \left[2(2+\epsilon)\left(v_a e^{\epsilon k y_a} e^{-\epsilon k \Delta y} - v_1\right)^2 - \epsilon v_1^2\right] e^{-4k\Delta y} + \mathcal{O}\left(e^{-12ky_a}, e^{-4k(\Delta y - y_a)} e^{-4k\Delta y}\right). \qquad (4.7)
\end{aligned}$$

To leading order, e^{-4ky_a}, this potential determines the equilibrium position for the axion brane,

$$ky_a = \frac{1}{\epsilon}\ln\frac{(2+\epsilon)v_0}{2v_a} \approx \frac{1}{\epsilon}\ln\frac{v_0}{v_a}. \qquad (4.8)$$

Eq. (4.7) illustrates why it is necessary first to consider the stabilization of the axion brane. The order $e^{-4ky_a}kM_5^3$ terms of the potential responsible for its stabilization are exponentially larger than the order $e^{-4k\Delta y}kM_3^5$ effects which stabilize the position of the IR brane in a Goldberger-Wise mechanism.

If we only have a free massive bulk scalar field as in the Goldberger-Wise mechanism [6], then the solution is already over-determined—for the second-order differential scalar field equation only two boundary conditions are required. We can quantify the fine tuning required for this system by determining how carefully the kink term on the axion brane needs to be adjusted to obtain natural size for the field at the IR brane. In general, the axion brane contains a potential term for the scalar field, $\mathcal{V}_a(\phi)$. We assume that this potential is relatively stiff, to enforce the boundary condition $\phi(y) = v_a$. Its presence produces a discontinuity in the y-derivative of the field at $y = y_a$,

$$\phi_+'(y_a) - \phi_-'(y_a) = M_5^{-3}\left.\frac{\delta\mathcal{V}_a}{\delta\phi}\right|_{\phi=v_a} \equiv M_5^{-3}\delta_\phi\mathcal{V}_a. \quad (4.9)$$

To learn how precisely $\delta_\phi\mathcal{V}_a$ must be tuned to obtain an natural $\mathcal{O}(1)$ size on the IR brane, we can alternately parameterize $\phi_+(y)$ in terms of v_a and $\delta_\phi\mathcal{V}_a$ and evaluate the result at the IR brane,

$$\begin{aligned}
\phi_+(\Delta y) &= \left[\frac{\delta_\phi\mathcal{V}_a}{2(2+\epsilon)kM_5^3} - v_0 e^{-\epsilon k y_a}\right] e^{(4+\epsilon)k(\Delta y - y_a)} \\
&\quad + \mathcal{O}(1). \qquad (4.10)
\end{aligned}$$

Although we see that $k^{-1}M_5^{-3}\delta_\phi\mathcal{V}_a \sim \mathcal{O}(1)$, it needs to be tuned to a precise value to within a factor of $e^{-4k(\Delta y - y_a)}$; otherwise the value of the field on the IR brane will be exponentially large. Introducing a mass scale for the scalar potential on the axion brane, $\mathcal{V}_a \sim M_a^4$ we see that M_a must be tuned to within $e^{-k(\Delta y - y_a)}$ of the value need to make the first term in Eq. (4.10) vanish.

Quite apart from producing effects large enough to distort the AdS$_5$ background, the value of $\phi(y)$ on the IR brane should be small if the mechanism to stabilize the intermediate brane is not to overwhelm any additional mechanism meant to stabilize the position of the IR brane.

V. THE NECESSITY OF A FINE TUNING FROM THE ADS/CFT PERSPECTIVE

From the previous section we observed that we must carefully tune some condition at the intermediate brane if its stabilization is not to prevent the stabilization or the appearance of the IR brane. This behavior has a natural interpretation from the perspective of the AdS/CFT correspondence.

In the AdS/CFT conjecture [11], the presence of the UV brane corresponds to an explicit breaking of a four dimensional conformal field theory (CFT) to a 4d CFT coupled to 4d gravity. If the AdS$_5$ space-time only contained an additional IR brane, we can understand its existence from the CFT side as follows [12]. A bulk scalar field of mass $m = k\sqrt{\epsilon(4+\epsilon)}$ corresponds to an almost marginal operator in the 4d CFT of dimension $4+\epsilon$ [13]. When this scalar field has a value v_1 on the IR brane, it spontaneously breaks the CFT, introducing a low energy mass scale. However since the operator is almost marginal it runs slowly, only becoming strongly interacting at an exponentially lower scale, $e^{-k\Delta y}M_4$.

The existence of a stabilized intermediate brane at $y = y_a$ requires that the CFT spontaneously breaks at some higher energy, $e^{-ky_a}M_4$. With the conformal symmetry broken thus at $e^{-ky_a}M_4 \gg e^{-k\Delta y}M_4 \sim$ TeV, we no longer have any mechanism to protect the IR brane fields from receiving corrections of the size of this intermediate scale other than explicitly finely tuning of the masses to within a fraction $e^{-k(\Delta y - y_a)}$, as was observed in the intermediate brane case in Eq. (4.10).

VI. CONCLUSIONS

In this article we have illustrated the difficulty in generating intermediate energy scales in the Randall-Sundrum model between the extremes associated with bulk gravity, $M_5 \sim k$, and with the physics of the IR brane, $e^{-k\Delta y}M_4$. The invisible axion solution to the strong CP provides a specific example for examining this problem since the experimental bounds on the scale of $U(1)_{\rm PQ}$ breaking lie well away from either of these extremes. If we express this scale in terms of a bulk redshift, $f \sim e^{-ky_a}M_4$, then the introduction of a phenomenologically acceptable axion requires a fine tuning of some parameter to the order $e^{-k(\Delta y - y_a)} \lesssim 10^{-6}$ which is not a significant improvement over simply setting $\bar\theta < 10^{-9}$.

We have restricted ourselves to fairly simple approaches for generating an intermediate scale since increasingly complicated mechanisms essentially replace a fine tuning of parameters with an unnaturally complex model. In the case of a bulk complex field with an arbitrarily shaped potential, the scalar field profile generally peaks near one or the other of the branes. For an intermediate brane, we found that in order to stabilize both this brane and the IR brane requires a precise choice for the size of the scalar field potential on the intermediate brane.

While the invisible axion is not an adequate solution of the strong CP problem in the Randall-Sundrum universe where we are concerned with understanding the origin of all large hierarchies of scales, the existence of extra dimensions may provide alternate methods for circumventing the complexity of the 4d QCD vacuum—for example by promoting the QCD gauge fields to 5d bulk fields [14]. Nevertheless, using this example to examine whether the Randall-Sundrum universe naturally admits intermediate scales provides a deeper understanding of the properties and the limitations of the scenario.

Acknowledgments

This work was supported in part by DOE grant DE-FG03-91-ER40682.

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